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A correction factor for Kac-Moody groups and *t*-deformed root multiplicities

Anna Puskás

University of Queensland

New Connections in Integrable Systems 1 October 2020



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Joint work with Dinakar Muthiah and Ian Whitehead; arXiv: 1806.05209, Mathematische Zeitschrift 296, pages 127–145 (2020)

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Macdonald's identity (1972, *The Poincaré series of a Coxeter group*):

$$\sum_{w \in W} w \left(\prod_{\alpha \in \Phi^+} \frac{1 - te^{\alpha}}{1 - e^{\alpha}} \right) = \sum_{w \in W} t^{\ell(w)}$$

W Weyl group, $\ell:W o \mathbb{Z}_{\geq 0}$ length function, Φ^+ positive roots. Kac-Moody root systems:

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Example: A_1 (\mathfrak{sl}_2)



Dynkin diagram: o, Cartan matrix: (2)



Macdonald's identity $\left(\, e^{lpha_1} \mapsto rac{x_1}{x_2} \,
ight)$

$$\frac{x_2 - t \cdot x_1}{x_2 - x_1} + \frac{x_1 - t \cdot x_2}{x_1 - x_2} = 1 + t$$

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Example: A_1 (\mathfrak{sl}_2)

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Example: A_2 (\mathfrak{sl}_3)



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Example: A_2 (\mathfrak{sl}_3)



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Example: A_2 (\mathfrak{sl}_3)

 $\sum_{w \in W} w \left(\prod_{\alpha \in \Phi^+} \frac{1 - t e^{\alpha}}{1 - e^{\alpha}} \right) = \sum_{w \in W} t^{\ell(w)}$ Dynkin diagram: • Cartan matrix: $\begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix}$ $\sum_{x_1} \left(\frac{x_2 - t \cdot x_1}{x_2 - x_1} \cdot \frac{x_3 - t \cdot x_2}{x_3 - x_2} \cdot \frac{x_3 - t \cdot x_1}{x_3 - x_1} \right) = 1 + 2t + 2t^2 + t^3$

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Example: A_2 (\mathfrak{sl}_3)

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$$\mathfrak{m} = \prod_{\lambda \in Q^+_{\mathrm{im}}} \prod_{n \ge 0} (1 - t^n e^{\lambda})^{-m(\lambda,n)}$$

where Q_{im}^+ positive imaginary root cone; and

$$m_{\lambda}(t) = \sum_{n \ge 0} m(\lambda, n) t^n$$

are polynomials with constant term: $m_{\lambda}(0) = \text{mult}(\lambda)$:

$$\mathfrak{m}|_{t=0} = \prod_{\alpha \in \Phi^+_{\mathrm{im}}} (1 - e^{\alpha})^{-\operatorname{\mathsf{mult}}(\alpha)}$$

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$$\Delta_{\mathrm{re}} = \prod_{\alpha \in \Phi_{\mathrm{re}}^+} (1 - e^{\alpha}), \ \Delta_{t,\mathrm{re}} = \prod_{\alpha \in \Phi_{\mathrm{re}}^+} (1 - te^{\alpha}), \ P(t) = \sum_{w \in W} t^{\ell(w)}$$

$$\mathfrak{m} \cdot \sum_{w \in W} w\left(\frac{\Delta_{t, \mathrm{re}}}{\Delta_{\mathrm{re}}}\right) = P(t)$$

$$\begin{split} \mathfrak{m}|_{t=0} \cdot rac{1}{\Delta_{\mathrm{re}}} \cdot \sum_{w \in W} (-1)^{\ell(w)} \cdot \prod_{lpha \in \Phi(w^{-1})} e^{lpha} = 1 \\ \mathfrak{m}|_{t=0} = \prod_{lpha \in \Phi_{\mathrm{im}}^+} (1 - e^{lpha})^{-\operatorname{mult}(lpha)} \end{split}$$

Remark: sometimes consider $lpha\mapsto -lpha$ (and omit ees).

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Formulae of *p*-adic Kac-Moody groups

$$S(\mathbb{1}_{K\pi^{\lambda}K}) = rac{1}{\mathfrak{m}} \cdot rac{t^{\langle
ho,\lambda
angle}}{P_{\lambda}(t)} \cdot \sum_{w\in W} w\left(e^{\lambda}rac{\Delta_{t,\mathrm{re}}}{\Delta_{\mathrm{re}}}
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Macdonald's formula for the spherical function

■ Generalizations: Braverman–Kazhdan–Patnaik (affine), Bardv-Panse–Gaussent–Rousseau (Kac–Moodv)

Taking a limit in λ , this converges to the Gindikin-Karpelevich formula, m persists. (Braverman–Garland–Kazhdan–Patnaik, Hébert, Ali)

Remark. Here \mathfrak{m} (not \mathfrak{m}') appears, factors corresponding to the multiplicities of imaginary roots were included in \mathfrak{m}

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Formulae of *p*-adic Kac-Moody groups

$$\mathcal{S}(\mathbb{1}_{K\pi^{\lambda}K}) = rac{1}{\mathfrak{m}} \cdot rac{t^{\langle
ho,\lambda
angle}}{P_{\lambda}(t)} \cdot \sum_{w\in W} w\left(e^{\lambda}rac{\Delta_{t,\mathrm{re}}}{\Delta_{\mathrm{re}}}
ight).$$

- Macdonald's formula for the spherical function
- Generalizations: Braverman–Kazhdan–Patnaik (affine), Bardy-Panse–Gaussent–Rousseau (Kac–Moody)

Taking a limit in λ , this converges to the Gindikin-Karpelevich formula, \mathfrak{m} persists. (Braverman–Garland–Kazhdan–Patnaik, Hébert, Ali)

Remark. Here \mathfrak{m} (not \mathfrak{m}') appears, factors corresponding to the multiplicities of imaginary roots were included in \mathfrak{m}

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Formulae of *p*-adic Kac-Moody groups, continued

 Casselman-Shalika formula for the spherical Whittaker function in affine type [Patnaik]

$$\mathcal{W}(\pi^{\lambda}) = t^{-\langle
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angle}\mathfrak{m}'\cdot\prod_{lpha\in\Phi^+}(1-te^{lpha})\chi_{\lambda}$$

Metaplectic analogue in Kac-Moody type [Patnaik, P.]

$$\mathcal{W}(\pi^{\lambda}) = \mathfrak{m}'_{R_n} \, \Delta_{R_n} \, \sum_{w \in W} (-1)^{\ell(w)} \left(\prod_{\widetilde{a} \in R_n(w)} e^{-\widetilde{a}}
ight) \, w \star e^{\lambda}$$

The factor m relates Hecke symmetrizers to Weyl symmetrizers.

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Macdonald's second proof of $1 \cdot \sum_{w \in W} w\left(\frac{\Delta_t}{\Delta}\right) = P(t)$ for finite W

Computation of the Betti numbers of a flag variety using Hodge theory.

- Right hand side: counting Schubert cells
- Left hand side: a computation of Dolbeault cohomology using localization at fixed points for the action of the maximal torus.
- The flag variety is smooth and projective Dolbeault cohomology is equal to Betti cohomology by the Hodge theorem.

Failure of $\mathfrak{m} = 1$ beyond finite type \Rightarrow Kac-Moody flag varieties are not smooth; they are homogeneous \Rightarrow everywhere singular.

Fishel-Grojnowski-Teleman explicitly compute the Dolbeault cohomology of the affine flag variety, prove *Strong Macdonald Conjecture*.

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$$\mathfrak{n}\sum_{w\in W}w\left(\frac{\Delta_{t,\mathrm{re}}}{\Delta_{\mathrm{re}}}\right)\stackrel{?}{=} P(t)$$

$$\Delta_{ ext{re}} = \prod_{lpha \in \Phi_{ ext{re}}^+} (1-e^lpha), \qquad \Delta_{t, ext{re}} = \prod_{lpha \in \Phi_{ ext{re}}^+} (1-te^lpha), \qquad {\mathcal P}(t) = \sum_{w \in W} t^{\ell(w)}$$

 $Q^{+} \supseteq Q^{+}_{im} \text{ cones graded by height;}$ Laurent series units on Q^{+} have form $ue^{\lambda_{0}} \prod_{\lambda \in Q^{+} \setminus \{0\}} \prod_{n} (1 - t^{n}e^{\lambda})^{m(\lambda,n)}$ $W \text{ acts on a multiplicative subset containing } \frac{\Delta_{tm}}{\Delta_{re}}$ $\sum_{w \in W} w \left(\frac{\Delta_{t,re}}{\Delta_{re}}\right) \text{ unit in } \mathbb{Z}[[t]][t^{-1}][[Q^{+}]], \text{ regular at } t = 0,$ constant coefficient P(t).

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Recall: we wish to define

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$$\sum_{w\in W} w\left(rac{\Delta_{t,\mathrm{re}}}{\Delta_{\mathrm{re}}}
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 and $P(t)$ unit in $\mathbb{Z}[[t]][t^{-1}][[Q^+]]$, regular at $t=0.$

We may define
$$\mathfrak{m}$$
 by $\mathfrak{m}\sum_{w\in W} w\left(rac{\Delta_{t,\mathrm{re}}}{\Delta_{\mathrm{re}}}
ight) = P(t)$

The factor \mathfrak{m} is Weyl-invariant and therefore supported on Q_{in}^+ .

Both m, m⁻¹ units in $\mathbb{Z}[t, t^{-1}][[Q^+]]$, regular at t = 0, constant coefficient 1.

$$\left. \left(\mathfrak{m}^{-1} \frac{\Delta_{\mathrm{re}}}{\Delta_{t,\mathrm{re}}} \right) \right|_{\mathcal{Q}^+_{\mathrm{im}}} = 1$$

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$$\left. \left(\mathfrak{m}^{-1} \frac{\Delta_{\mathrm{re}}}{\Delta_{t,\mathrm{re}}} \right) \right|_{Q^+_{\mathrm{im}}} = 1.$$

In the affine case, this implies "constant term property"

$$\left(\frac{\Delta_{\rm re}}{\Delta_{t,\rm re}}\right)\Big|_{Q^+_{\rm im}} = \mathfrak{m}$$

In the Kac-Moody case, this is not true!

$$\operatorname{Supp}(\mathfrak{a})\subseteq Q^+_{\operatorname{im}}
eq (\mathfrak{a}\cdot\mathfrak{b})|_{Q^+_{\operatorname{im}}}=\mathfrak{a}\cdot(\mathfrak{b})|_{Q^+_{\operatorname{im}}}$$

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$$\operatorname{Supp}(\mathfrak{a}) \subseteq Q_{\operatorname{im}}^+ \not\Rightarrow (\mathfrak{a} \cdot \mathfrak{b})|_{Q_{\operatorname{im}}^+} = \mathfrak{a} \cdot (\mathfrak{b})|_{Q_{\operatorname{im}}^+}$$

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$$\left. \left(\mathfrak{m}^{-1} rac{\Delta_{\mathrm{re}}}{\Delta_{t,\mathrm{re}}}
ight)
ight|_{Q^+_{\mathrm{im}}} = 1.$$

In the affine case, this implies "constant term property"

$$\left(rac{\Delta_{\mathrm{re}}}{\Delta_{t,\mathrm{re}}}
ight)
ight|_{Q^+_{\mathrm{im}}}=\mathfrak{m}$$

In the Kac-Moody case, this is not true!

$$\mathsf{Supp}(\mathfrak{a})\subseteq Q^+_{\mathrm{im}}\not\Rightarrow (\mathfrak{a}\cdot\mathfrak{b})|_{Q^+_{\mathrm{im}}}=\mathfrak{a}\cdot(\mathfrak{b})|_{Q^+_{\mathrm{im}}}$$

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$$\mathfrak{m} = \prod_{i=1}^{\infty} \left(\left(\frac{1-t \cdot e^{i \cdot \delta}}{1-e^{i \cdot \delta}} \right)^r \cdot \prod_{j=1}^r \frac{1-t^{m_j} \cdot e^{i \cdot \delta}}{1-t^{m_j+1} \cdot e^{i \cdot \delta}} \right)$$

$$\mathfrak{m} = \prod_{i=1}^{\infty} \prod_{j=1}^{r} \left(\prod_{k=1}^{m_j} \frac{(1-t^k \cdot e^{i \cdot \delta})^2}{(1-t^{k-1}e^{i \cdot \delta})(1-t^{k+1}e^{i \cdot \delta})} \right)$$

$$-m_{l\cdot\delta}(t)=\sum_{j=1}^r \left(\sum_{k=1}^{m_j}t^{k-1}\cdot(-1+2t-t^2)
ight)=-(1-t)^2\cdot\sum_{j=1}^rrac{t^{m_j}-1}{t-1}$$

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Preview and Background	Motivation	Definition and properties	Affine case	Results beyond affine type	Further
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Cherednik's solution of Macdonald's Constant Term Conjecture: \mathfrak{m} is known for Φ of affine type.

$$\mathfrak{m} = \prod_{i=1}^{\infty} \left(\left(\frac{1-t \cdot e^{i \cdot \delta}}{1-e^{i \cdot \delta}} \right)^r \cdot \prod_{j=1}^r \frac{1-t^{m_j} \cdot e^{i \cdot \delta}}{1-t^{m_j+1} \cdot e^{i \cdot \delta}} \right)$$

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Cherednik's solution of Macdonald's Constant Term Conjecture: ${\mathfrak m}$ is known for Φ of affine type.

For untwisted, simply laced affine types:

$$\mathfrak{m} = \prod_{i=1}^{\infty} \left(\left(\frac{1-t \cdot e^{i \cdot \delta}}{1-e^{i \cdot \delta}} \right)^r \cdot \prod_{j=1}^r \frac{1-t^{m_j} \cdot e^{i \cdot \delta}}{1-t^{m_j+1} \cdot e^{i \cdot \delta}} \right)$$

where r is the rank, m_j exponents of underlying finite-dimensional root system, δ the minimal imaginary root.

$$\mathfrak{m} = \prod_{i=1}^{\infty} \prod_{j=1}^{r} \left(\prod_{k=1}^{m_j} \frac{(1-t^k \cdot e^{i \cdot \delta})^2}{(1-t^{k-1}e^{i \cdot \delta})(1-t^{k+1}e^{i \cdot \delta})} \right)$$

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$$\mathfrak{m} = \prod_{\lambda \in \mathcal{Q}_{\mathrm{im}}^+} \left(\prod_{\beta \in \mathcal{S}(\lambda)} \frac{(1 - t^{\mathrm{ht}(\beta)} e^{\lambda})^2}{(1 - t^{\mathrm{ht}(\beta) - 1} e^{\lambda})(1 - t^{\mathrm{ht}(\beta) + 1} e^{\lambda})} \right)$$

where

$$S(\lambda) = \{ \beta \in Q_{\mathrm{fin}}^+ \mid \beta + \lambda \in \Phi_{\mathrm{re}} \},$$

 Q_{fin}^+ is a root lattice corresponding to a finite root subsystem $\Phi_{\text{fin}} \subseteq \Phi$ determined by omitting an appropriate simple root.

$$m_{\lambda} = (1-t)^2 \cdot \sum_{\beta \in S(\lambda)} t^{\operatorname{ht}(\beta)-1}$$

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Generalized Petersen algorithm

We wish to write

$$\mathfrak{m} = \prod_{\lambda \in \mathcal{Q}^+_{\mathrm{im}}} \prod_{n=0}^{N_{\lambda}} (1 - t^n e^{\lambda})^{-m(\lambda,n)}$$

starting from

$$\left(\mathfrak{m}^{-1} \frac{\Delta_{\mathrm{re}}}{\Delta_{t,\mathrm{re}}}\right)\Big|_{Q^+_{\mathrm{im}}} = 1$$

- \blacksquare power series inverse with respect to Q_{im}^+
- by induction on height
- algorithm polynomial in height
- generalization of the Petersen algorithm for $mult(\lambda)$
- suffices to compute for one λ per W-orbit, i.e. on antidominant cone

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$$\mathfrak{m} = \prod_{\lambda \in Q_{\mathrm{im}}^+} \prod_{n=0}^{N_{\lambda}} (1 - t^n e^{\lambda})^{-m(\lambda,n)}$$

Set $N_{\alpha} = 1$, $m(\alpha, 0) = 1$, $m(\alpha, 1) = -1$ for $\alpha \in \Phi_{re}$; $m(\lambda, n) = 0$ if λ is not an imaginary vector or a real root.

$$\left(\mathfrak{m}^{-1} \frac{\Delta_{\mathrm{re}}}{\Delta_{t,\mathrm{re}}} \right) \Big|_{Q_{\mathrm{im}}^{+}} = 1; \quad \mathfrak{m}^{-1} \frac{\Delta_{\mathrm{re}}}{\Delta_{t,\mathrm{re}}} = \prod_{\lambda \in Q^{+}} \prod_{n=0}^{N_{\lambda}} (1 - t^{n} e^{\lambda})^{m(\lambda,n)} = \prod_{\lambda \in Q^{+}} \mathfrak{m}_{\lambda}^{-1}$$
$$\mathfrak{m}_{\lambda}^{-1} \cdot \prod_{\substack{\mu \in Q^{+} \\ \operatorname{ht}(\mu) < \operatorname{ht}(\lambda)}} \mathfrak{m}_{\mu}^{-1} \Big|_{\lambda} = 0$$

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$$\mathfrak{m} = \prod_{\lambda \in \mathcal{Q}^+_{\mathrm{im}}} \prod_{n=0}^{N_{\lambda}} (1 - t^n e^{\lambda})^{-m(\lambda,n)}$$

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Set $\mathfrak{m}_0 = 1$. Assume $\operatorname{ht}(\lambda) > 0$, and \mathfrak{m}_{μ} known for $\operatorname{ht}(\mu) < \operatorname{ht}(\lambda)$ $\mu \in Q_{\operatorname{im}}^+$: from previous steps of the induction $\mu \notin Q_{\operatorname{im}}^+$: by computing real roots up to $\operatorname{ht}(\lambda)$ The coefficient of e^{λ} in $\prod_{\substack{\mu \in Q^+ \\ \operatorname{ht}(\mu) < \operatorname{ht}(\lambda)}} \mathfrak{m}_{\mu}^{-1}$ is a polynomial in t := $\mathfrak{m}_{\mu}(\lambda, 0) + \mathfrak{m}_{\mu}(\lambda, 1)t + \cdots + \mathfrak{m}_{\mu}(\lambda, N_{\lambda})t^{N_{\lambda}}$

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The coefficient of
$$e^{\lambda}$$
 in $\prod_{\substack{\mu \in Q^+ \\ ht(\mu) < ht(\lambda)}} m_{\mu}^{-1}$ is a polynomial in $t := m(\lambda, 0) + m(\lambda, 1)t + \dots + m(\lambda, N_{\lambda})t^{N_{\lambda}}$

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$$\mathfrak{m}_{\lambda}^{-1} \cdot \prod_{\substack{\mu \in Q^+ \\ \mathsf{ht}(\mu) < \mathsf{ht}(\lambda)}} \mathfrak{m}_{\mu}^{-1} \bigg|_{\lambda} = \prod_{n=0}^{N_{\lambda}} (1 - t^n e^{\lambda})^{m(\lambda, n)} \cdot \prod_{\substack{\mu \in Q^+ \\ \mathsf{ht}(\mu) < \mathsf{ht}(\lambda)}} \mathfrak{m}_{\mu}^{-1} \bigg|_{\lambda} = 0$$

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$$\mathfrak{m}_{\lambda}^{-1} \cdot \prod_{\substack{\mu \in Q^+ \\ \mathsf{ht}(\mu) < \mathsf{ht}(\lambda)}} \mathfrak{m}_{\mu}^{-1} \bigg|_{\lambda} = \prod_{n=0}^{N_{\lambda}} (1 - t^n e^{\lambda})^{m(\lambda, n)} \cdot \prod_{\substack{\mu \in Q^+ \\ \mathsf{ht}(\mu) < \mathsf{ht}(\lambda)}} \mathfrak{m}_{\mu}^{-1} \bigg|_{\lambda} = 0$$

Set $\mathfrak{m}_0 = 1$. Assume $\operatorname{ht}(\lambda) > 0$, and \mathfrak{m}_{μ} known for $\operatorname{ht}(\mu) < \operatorname{ht}(\lambda)$ $\mu \in Q^+_{\operatorname{im}}$: from previous steps of the induction $\mu \notin Q^+_{\operatorname{im}}$: by computing real roots up to $\operatorname{ht}(\lambda)$

The coefficient of
$$e^{\lambda}$$
 in $\prod_{\substack{\mu \in Q^+ \\ \operatorname{ht}(\mu) < \operatorname{ht}(\lambda)}} \mathfrak{m}_{\mu}^{-1}$ is a polynomial in $t := m(\lambda, 0) + m(\lambda, 1)t + \cdots + m(\lambda, N_{\lambda})t^{N_{\lambda}}$

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$$\mathfrak{m}_{\lambda}^{-1} \cdot \prod_{\substack{\mu \in Q^+ \\ \mathsf{ht}(\mu) < \mathsf{ht}(\lambda)}} \mathfrak{m}_{\mu}^{-1} \bigg|_{\lambda} = \prod_{n=0}^{N_{\lambda}} (1 - t^n e^{\lambda})^{m(\lambda, n)} \cdot \prod_{\substack{\mu \in Q^+ \\ \mathsf{ht}(\mu) < \mathsf{ht}(\lambda)}} \mathfrak{m}_{\mu}^{-1} \bigg|_{\lambda} = 0$$

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Generalized Berman-Moody formula

Theorem [Muthiah-P-Whitehead] For all $\lambda \in Q^+$, we have:

$$m_{\lambda}(t) = \sum_{\kappa \mid \lambda} \mu\left(\lambda/\kappa\right) \left(\frac{\lambda}{\kappa}\right)^{-1} \sum_{\underline{\kappa} \in \mathsf{Par}(\kappa)} (-1)^{|\underline{\kappa}|} \frac{B(\underline{\kappa})}{|\underline{\kappa}|} \prod_{i=1}^{|\underline{\kappa}|} P_{\kappa_i}(t^{\lambda/\kappa})$$

For t = 0 recovers the Berman-Moody formula for mult(λ) = m_λ(0)
λ, κ ∈ Q⁺, λ = k ⋅ κ, then κ|λ, λ/κ = k ∈ Z, μ(λ/κ) Möbius function
Par(λ) vector partitions of λ, |κ|, B(κ)

• $P_{\kappa_i}(t^{\lambda/\kappa})=P_{\kappa_i}(t^k)$ given in terms of Kostant partitions of $\kappa_i\in Q^+.$

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 logarithm, differential operator $\sum_{i} e^{\alpha_{i}} \frac{\partial}{\partial e^{\alpha_{i}}}$, Möbius transform

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$$= \frac{\Delta_{re}}{\mathfrak{m}\Delta_{tre}} \log_{re} \operatorname{logarithm}, \operatorname{differential operator} \sum_{i} e^{\alpha_{i}} \frac{\partial}{\partial e^{\alpha_{i}}}, \operatorname{Möbius transform}$$

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Theorem [Muthiah-P-Whitehead] For $\lambda \in Q_{im}^+$, $m_\lambda(t) \neq 0 \Leftrightarrow \lambda \in \Phi_{im}$. If $\Phi_1 \subseteq \Phi$ root subsystem, $Q_1 \subseteq Q$, \mathfrak{m}_1 , \mathfrak{m} ; then $\mathfrak{m}|_{Q_1} = \mathfrak{m}_1$. If $\Phi_1, \Phi_2 \subseteq \Phi$, simple roots $\Delta_1 \perp \Delta_2$, then $\mathfrak{m} = \mathfrak{m}_1\mathfrak{m}_2$. If $\lambda \in Q_{im}^+ \setminus \Phi_{im}$ antidominant, then $\operatorname{Supp}_\Delta \lambda$ disconnected. **Theorem** [Muthiah-P-Whitehead] For $\lambda \in Q_{im}^+$, $(1-t)^2 | m_\lambda(t)$. Use Generalized Berman-Moody formula Observation: $\lambda \in Q_{im}^+$ as sum over Φ_{re} has at least two terms. For any $\mu \in Q^+$, $\mu \neq 0$: $P_\mu(1) = 0$

$m_{\lambda}(t) = \sum_{\kappa \mid \lambda} \mu\left(\lambda/\kappa\right) \left(\frac{\lambda}{\kappa}\right)^{-1} \sum_{\underline{\kappa} \in \mathsf{Par}(\kappa)} (-1)^{|\underline{\kappa}|} \frac{B(\underline{\kappa})}{|\underline{\kappa}|} \prod_{i=1}^{|\underline{\kappa}|} P_{\kappa_i}(t^{\lambda/\kappa})$

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Examples

An illustration...

$$\chi_{\lambda}(t) = \frac{m_{\lambda}(t)}{(1-t)^2} = \frac{\sum_{i=0}^{N_{\lambda}} m(\lambda, n) \cdot t^n}{(1-t)^2}$$

Using the Generalized Petersen algorithm, compute this for the hyperbolic root systems with Cartan matrices

$$\begin{bmatrix} 2 & -3 \\ -2 & 2 \end{bmatrix}, \begin{bmatrix} 2 & -3 \\ -3 & 2 \end{bmatrix} \text{ and } \begin{bmatrix} 2 & -2 & 0 \\ -2 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

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Using the Generalized Petersen algorithm, compute this for the hyperbolic root systems with Cartan matrices

$$\begin{bmatrix} 2 & -3 \\ -2 & 2 \end{bmatrix}, \begin{bmatrix} 2 & -3 \\ -3 & 2 \end{bmatrix} \text{ and } \begin{bmatrix} 2 & -2 & 0 \\ -2 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}.$$

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Examples

 $\chi_{\lambda}(t)$, Cartan matrix $\begin{vmatrix} 2 & -3 \\ -2 & 2 \end{vmatrix}$ $\lambda \qquad \chi_{\lambda}(t)$ (1,1) 1 (2,2) -t+1(3,2) $t^2 + 0t + 2$ $(3,3) -t^3 - 2t + 2$ $(4,3) \quad t^4 - t^3 + 2t^2 - 3t + 3$ (4, 4) $-t^5 + t^4 - 2t^3 + 3t^2 - 6t + 3$ (5, 4) $t^6 - 2t^5 + 4t^4 - 6t^3 + 9t^2 - 9t + 6$ $(5,5) \quad -t^7 + t^6 - 4t^5 + 6t^4 - 10t^3 + 13t^2 - 13t + 7$ $(6,4) t^6 - 4t^5 + 5t^4 - 8t^3 + 11t^2 - 13t + 6$ (10, 9) $t^{16} - 7t^{15} + 29t^{14} - 91t^{13} + 248t^{12} - 584t^{11} + 1197t^{10} -$ $2170t^9 + 3505t^8 - 5039t^7 + 6437t^6 - 7253t^5 + 7042t^4 -$ $5618t^3 + 3405t^2 - 1372t + 272$

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Examples

 $\chi_{\lambda}(t)$, Cartan matrix $\begin{vmatrix} 2 & -3 \\ -3 & 2 \end{vmatrix}$ $egin{array}{ccc} \lambda & \chi_\lambda(t) \ (1,1) & 1 \end{array}$ (2,2) -2t+1(2,3) $t^2 - t + 2$ (3,3) $-2t^3+3t^2-4t+3$ (3,4) $t^4 - 3t^3 + 6t^2 - 6t + 4$ (4, 4) $-2t^5 + 7t^4 - 12t^3 + 17t^2 - 16t + 6$ (4,5) $t^6 - 5t^5 + 15t^4 - 26t^3 + 30t^2 - 23t + 9$ $(4,6) t^6 - 8t^5 + 19t^4 - 31t^3 + 36t^2 - 28t + 9$ $-2t^7 + 9t^6 - 30t^5 + 58t^4 - 82t^3 + 77t^2 - 50t + 16$ (5, 5). (10, 9) $t^{16} - 15t^{15} + 135t^{14} - 811t^{13} + 3535t^{12} - 11729t^{11} +$ $30615t^{10} - 64282t^9 + 110096t^8 - 154852t^7 + 178868t^6 -$ $168420t^{5} + 127110t^{4} - 74539t^{3} + 32094t^{2} - 9070t + 1267$

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Further Questions and Remarks					

Conjecture The polynomials χ_{λ} have alternating sign coefficients in rank two hyperbolic type.

Problem Interpret all coefficients of χ_{λ} in terms of the Kac-Moody Lie algebra.

Problem Give upper bounds for the degree and coefficients of $\chi_{\lambda}(t)$.

Question Relationship of $m_{\lambda}(t)$ and Kac polynomials?

Question What is the Dolbeault cohomology of Kac-Moody flag varieties? (A two-parameter generalization of m.)

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Thank you!

