Junichi Shiraishi (The University of Tokyo)

Title: Some conjectures concerning non-stationary Ruijsenaars functions

Abstract: My talk is based on the collaboration with Edwin Langmann and Masatoshi Noumi. I define a certain formal power series $f^{\widehat{\mathfrak{gl}}_N}(x,p|s,\kappa|q,t)$ (called the non-stationary Ruijsenaars functions), show some basic properties, and present several conjectures [S][LNS].

The asymptotically free solution [NS] (which I call $f^{\mathfrak{gl}_N}(x|s|q,t)$) to the Macdonald difference equations of type A give the Euler characteristics of the Laumon spaces [BFS]. In the same way, the non-stationary Ruijsenaars functions $f^{\widehat{\mathfrak{gl}}_N}(x,p|s,\kappa|q,t)$ represent the the Euler characteristics of the affine Laumon spaces [FFNR]. Based on the screened vertex operators associated with the affine screening operators, $f^{\widehat{\mathfrak{gl}}_N}(x,p|s,\kappa|q,t)$ can be obtained as a certain correlation functions of the screened vertex operators.

The Schur limit (i.e. $t \to q$) can be stated as follows. When the parameters s and κ are suitably chosen, the limit $t \to q$ of $f^{\widehat{\mathfrak{gl}}_N}(x,p|s,\kappa|q,q/t)$ gives us the dominant integrable characters of $\widehat{\mathfrak{sl}}_N$ multiplied by $1/(p^N;p^N)_\infty$ (*i.e.* the $\widehat{\mathfrak{gl}}_1$ character).

Some basic conjectures concerning the non-stationary Ruijsenaars function $f^{\widehat{\mathfrak{gl}}_N}(x,p|s,\kappa|q,t)$ are as follows.

(1) We have the duality conjectures for the suitably normalized version $\varphi^{\widehat{\mathfrak{gl}}_N}(x,p|s,\kappa|q,t)$: $\varphi^{\widehat{\mathfrak{gl}}_N}(x,p|s,\kappa|q,t) = \varphi^{\widehat{\mathfrak{gl}}_N}(s,\kappa|x,p|q,t)$ (bispectral duality) and $\varphi^{\widehat{\mathfrak{gl}}_N}(x,p|s,\kappa|q,t) = \varphi^{\widehat{\mathfrak{gl}}_N}(x,p|s,\kappa|q,q/t)$ (Poincare' duality).

(2) An important special case appears in the very singular (essentially singular) limit $\kappa \to 1$. It is expected that one can normalize $f^{\widehat{\mathfrak{gl}}_N}(x,p|s,\kappa|q,t)$ in such a way that the limit $\kappa \to 1$ exists, and the limit $f^{\mathrm{st.}\,\widehat{\mathfrak{gl}}_N}(x,p|s|q,t)$ gives us the eigenfunction of the elliptic Ruijsenaars operator.

(3) To explore some candidates of eigenvalue equations which $f^{\widehat{\mathfrak{gl}}_N}(x,p|s,\kappa|q,t)$ should satisfy, I introduce a new operator $\mathcal{T}^{\widehat{\mathfrak{gl}}_N}(x,p|q,t,\kappa)$. This is obtained by extending a similar operator $\mathcal{T}^{\mathfrak{gl}_N}(x,p|q,t)$ having $f^{\mathfrak{gl}_N}(x|s|q,t)$ as the eigensunctions. Both $\mathcal{T}^{\mathfrak{gl}_N}(x,p|q,t)$ and $\mathcal{T}^{\widehat{\mathfrak{gl}}_N}(x,p|q,t,\kappa)$ are operators containing $q^{\frac{1}{2}\Delta}$ (where Δ is the odinary Laplacian), and seem new object (even in the Macdonald case).

[**BFS**] A. Braverman, M. Finkelberg and J. Shiraishi, Macdonald polynomials, Laumon spaces and perverse coherent sheaves, Perspectives in representation theory, 23–41, Contemp. Math., **610**, Amer. Math. Soc., Providence, RI, 2014.

[**FFNR**] B. Feigin, M. Finkelberg, A. Negut and L. Rybnikov, Yangians and cohomology ring of Laumon spaces, Sel. Math. New. Ser. (2011) **17**:573-607, DOI 10.1007/s00029-011-0059-x.

[**LNS**] E. Langmann, M. Noumi and J. Shiraishi, Basic properties of non-stationary Ruijsenaars functions, `arXiv:2006.07171`.

[**NS**] M. Noumi and J. Shiraishi, A direct approach to the bispectral problem for the Ruijsenaars-Macdonald q-difference operators, `arXiv:1206.5364`.

[**S**] J. Shiraishi, Affine Screening Operators, Affine Laumon Spaces, and Conjectures Concerning Non-Stationary Ruijsenaars Functions, J. of Int. Systems **4** (2019), xyz010.