Learning Seminar on Kazhdan–Lusztig Theory

POSTECH (2026-2)

Overview

Kazhdan–Lusztig (KL) polynomials are certain polynomials attached to a Weyl group or a Coxeter group (or more precisely their Hecke algebra) that have a very simple, purely combinatorial definition. Despite their simplicity, they play a major role in representation theory, geometry and combinatorics:

- they keep track of relations between standard modules and irreducible modules in many non-semisimple categories of interest in representation theory (starting with category $\mathcal{O}$, see the original Kazhdan–Lusztig conjecture).
- they encode information about the local intersection cohomology of Schubert varieties and have other geometrical incarnations.
- exhibit interesting combinatorial properties due to their positivity (ex combinatorial formulas for Kostka Foulkes polynomials)

This seminar will study Kazhdan–Lusztig (KL) theory and its connections to the areas above.

Prerequisites. Graduate students with a background in algebra. It is useful if you know some representation theory (say semisimple Lie algebra over $\mathbb{C}$, root systems etc.), but one may follow many parts of the seminar without this.

Format. Weekly meetings of 2–2.25 hours each. All talks (besides the first) will be given by students. All students are welcome to meet with the coordinator and discuss their talk (or the seminar) in detail. There are some homework problem spread throughout, students are encouraged to work on them in groups.

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Website. website

Structure.

- Session 0: Introduction to the seminar.
- Part I (Sessions 1–2): Coxeter combinatorics, the Hecke algebra, and the KL basis.
- Part II (Session 3): Category $\mathcal{O}$ and the statement of the Kazhdan–Lusztig conjecture.
- Part III (Sessions 4–5): Affine and parabolic/singular KL theory.
- Part IV (Sessions 6–7): Geometric origins — Schubert varieties, intersection cohomology, and Beilinson–Bernstein localization.
- Part V (Session 8): Soergel bimodules and the proof of positivity.
- Part VI (Sessions 9–10): The p-adic Kazhdan–Lusztig hypothesis for $GL_n(F)$ (Zelevinsky).
- Part VII (Sessions 11–12): KL-type polynomials in finite-dimensional representations of quantum affine algebras (Nakajima).

Schedule

The schedule is approximate. We might go faster or (more likely) slower than expected. There might also be changes based on the interest of the audience.

Session	Date	Title	Speaker
0	09/07	Introduction to the Seminar	Valentin
1	09/14	Coxeter Systems, the Bruhat Order, and the Iwahori–Hecke Algebra	
2	09/21	The Kazhdan–Lusztig Basis and Polynomials	
3		Category $\mathcal{O}$ and the Algebraic KL Conjecture	
4		Affine Weyl Groups and Affine Kazhdan–Lusztig Polynomials	
5		Parabolic and singular KL Polynomials, Kostka Foulkes polynomials	
6		Flag Varieties, Schubert Varieties, and Their Singularities	
7		Intersection Cohomology	
8		Soergel Bimodules and the Proof of Positivity	
9		The Zelevinsky Classification of Representations of $GL_n(F)$	
10		Zelevinsky's p-adic Kazhdan–Lusztig Hypothesis and Its Proof	
11		Quiver Varieties and Finite-Dim. Reps. of Quantum Affine Algebras	
12		q, t-Characters and KL-Type Polynomials for Quantum Affine Algebras	
13		Advancing mathematics by guiding human intuition with AI	